

# Ecosystems

Models of population  
dynamics

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# Simple starting point for ecosystem modelling

Population modelling

Basis of all ecology

What controls and regulates population growth and size?

How can humans exploit populations of animals and plants sustainably?

# What can go wrong?

Human activities influence almost all natural populations

When we do not understand population dynamics we may observe unwanted

- Extinction
- Invasion and over population
- Non sustainable use
- Population imbalances (boom and bust)

# How do we study populations?

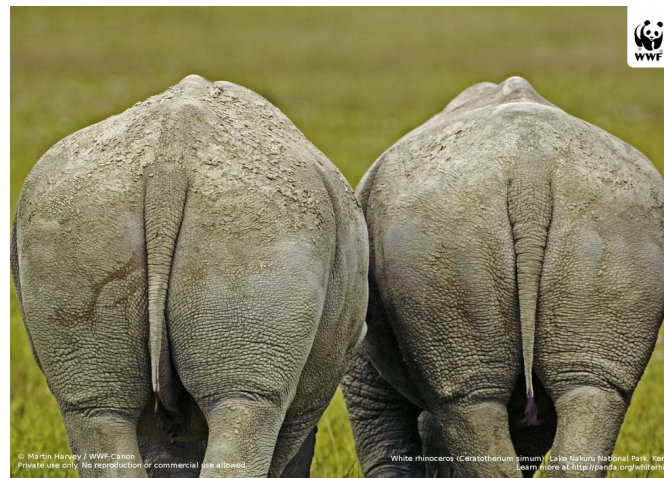
Visible populations of conservation concern are continuously monitored.

For example, Wildebeest numbers in the Serengeti



# How do we study populations?

Populations change through births, deaths and migration



# How do we study populations?

Births deaths and migration are known quantities for some populations



# How do we study populations?

The populations of most organisms cannot be precisely monitored

We usually have to infer population size and produce indirect estimates.

We also frequently need to make projections regarding population dynamics.

Mathematical modelling allows us to .....

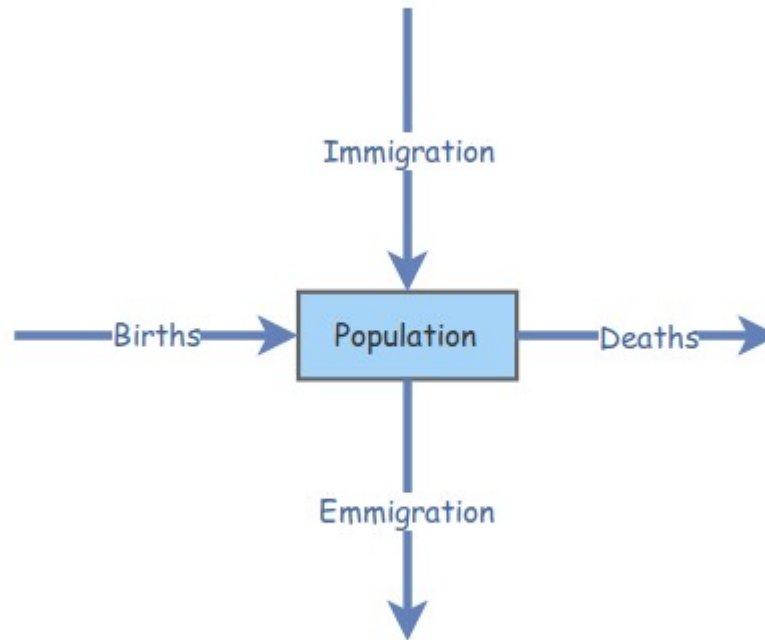
1. Estimate unmeasurable population parameters
2. Predict population change over time
3. Understand processes responsible for change

# Types of models

- 1) Aggregated population models using differential equations – make simple assumptions regarding births and deaths.
- 2) Disaggregated models (matrix models) - take into account population structure
- 3) Individual (agent) based models -take into account variability at the level of single organisms
- 4) Spatially explicit models – take into account effects of habitat patch size and connectivity
- 5) More complex simulation models - take into account interactions at the community level

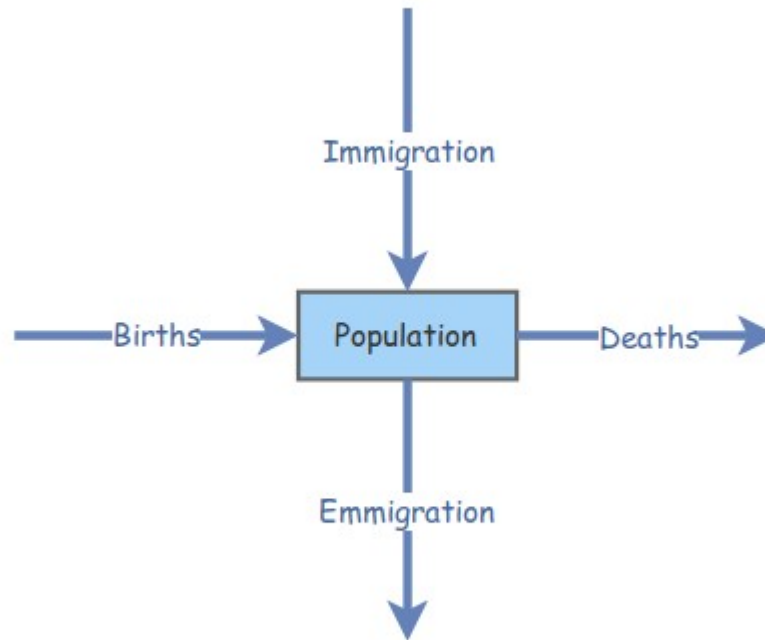


# Aggregated population model



$$N_{t+1} = N_t + Births_t + In_t - Deaths_t - Out_t$$

# Differential equation



$$\frac{dN}{dt} = \textit{Births} + \textit{In} - \textit{Deaths} - \textit{Out}$$

“The population size is the integrated result of births, deaths and migration with respect to to time”

# Ignoring migration



$$\frac{dN}{dt} = \textit{Births} - \textit{Deaths}$$

“The population size is the integrated result of births minus deaths with respect to to time”

# Population stability

A population can only be stable if births exactly equal deaths.

This is highly unlikely.

In reality all populations undergo natural fluctuations in size

We therefore attempt to model and understand **population dynamics**

# Building a simple model

Let's first assume that a population (call them rabbits) consisting of 100 individuals has a fixed number of births each year (let's say 100).

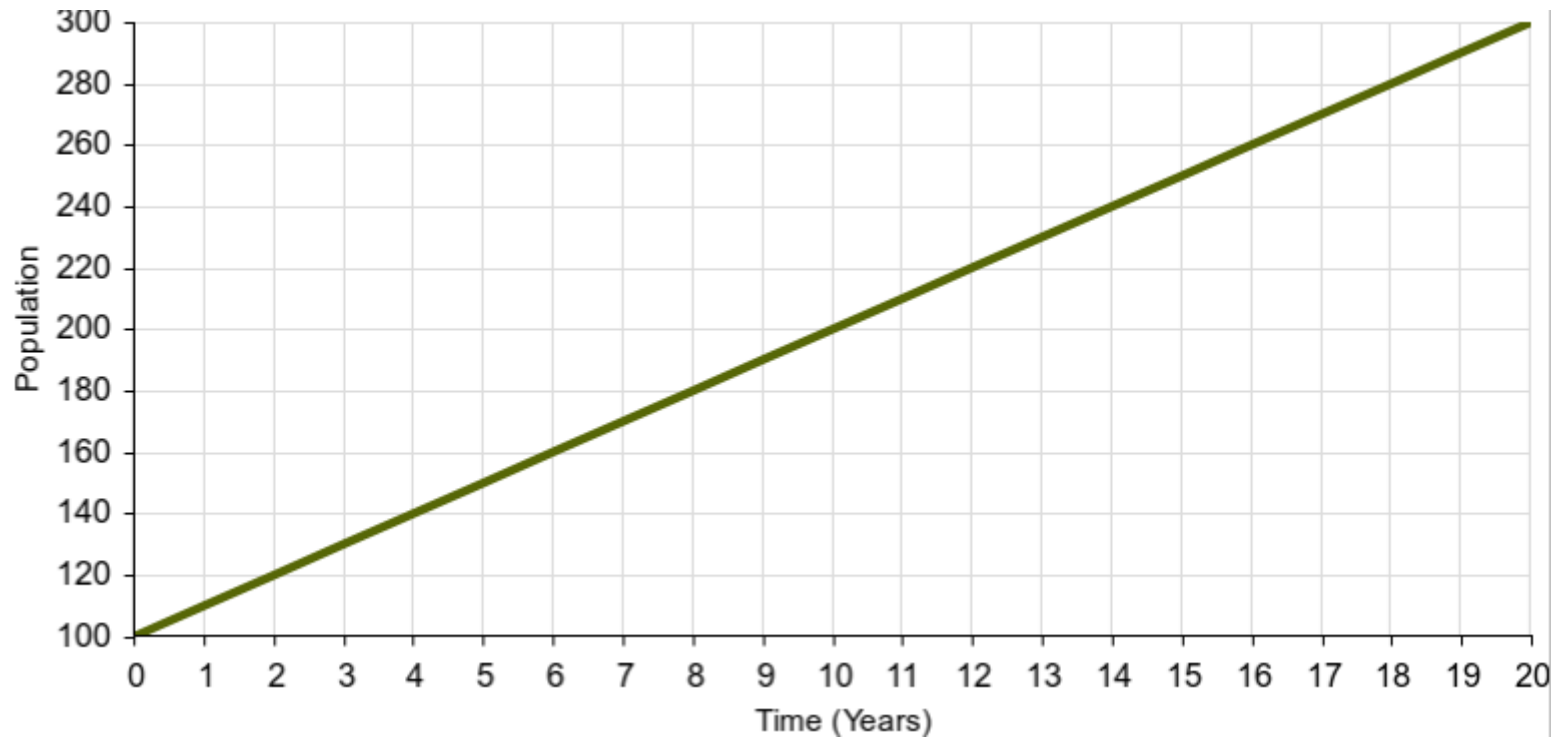
Let's set a fixed number of deaths (say 90)

So after one year  $N_{t+1} = 100 + 100 - 90 = 110$



# Building a simple model

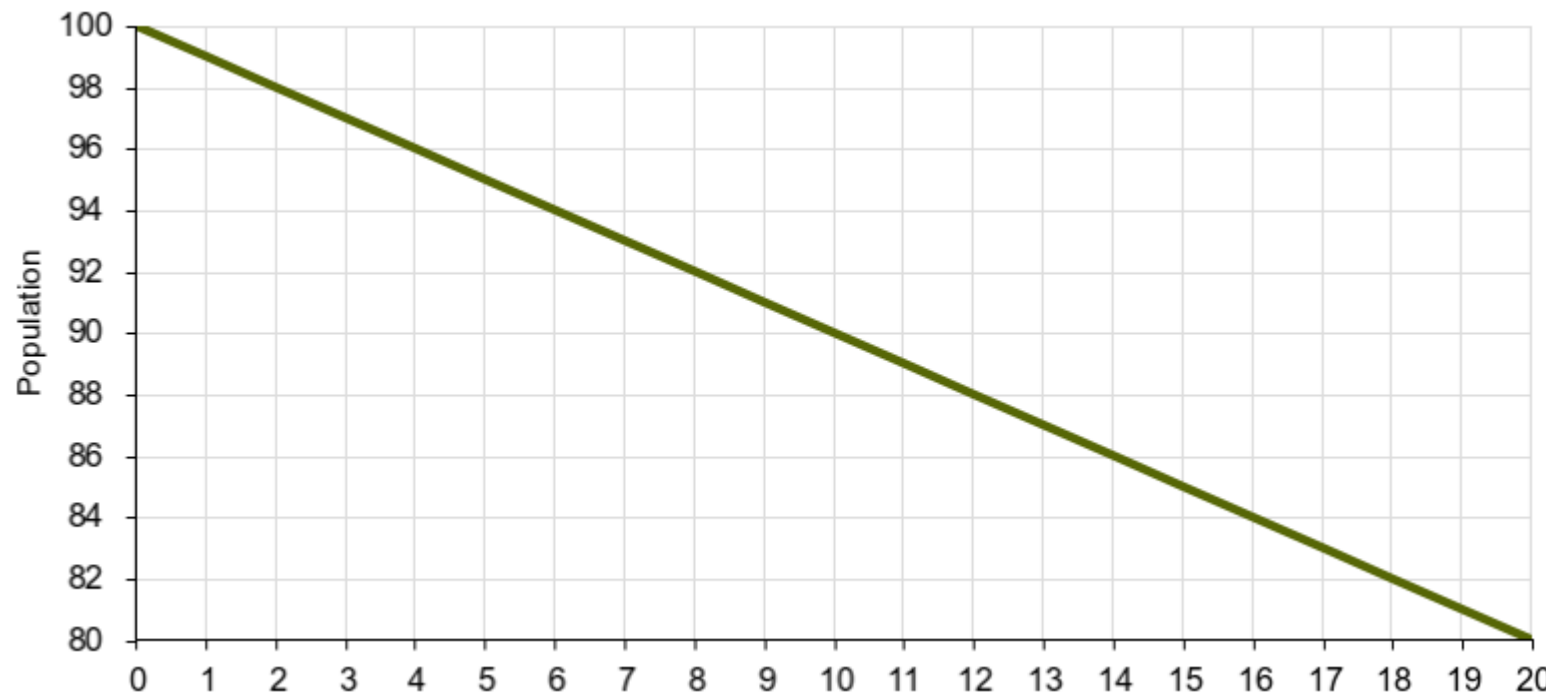
What happens if this continues year after year?



# Building a simple model

What happens if number of deaths always are greater than births?

$$N_{t+1} = 90 - 89$$



# Building a simple model

This is NOT a suitable model!

Births and deaths **cannot** be a constant number!

We need (at least) to think about birth and death rates.

$$r = \textit{BirthRate} - \textit{DeathRate}$$

$$\frac{dN}{dt} = rN$$



# What does this imply?

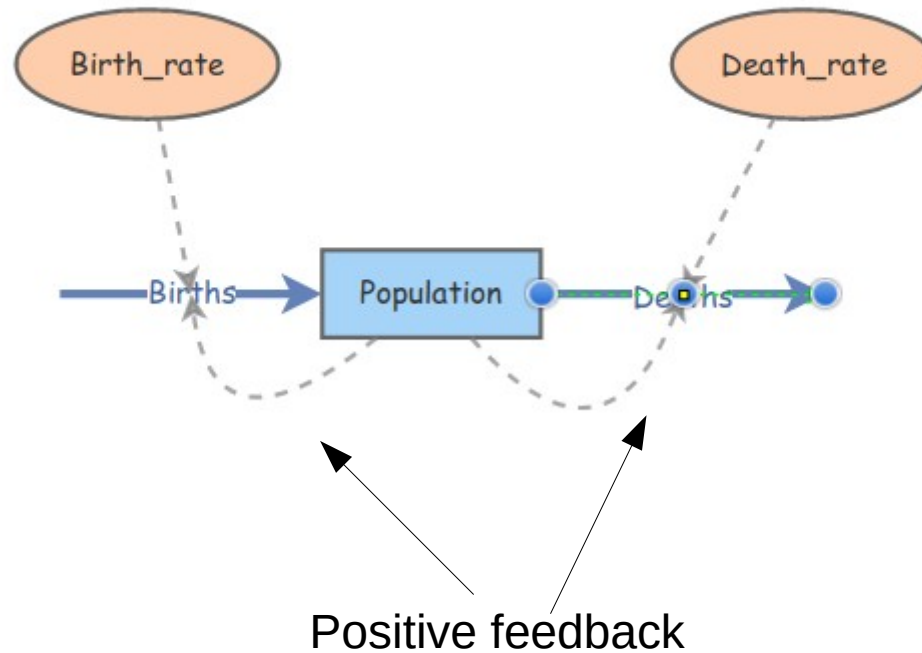
Stability only occurs when  $r = 0$

If  $r$  is positive we get exponential growth.

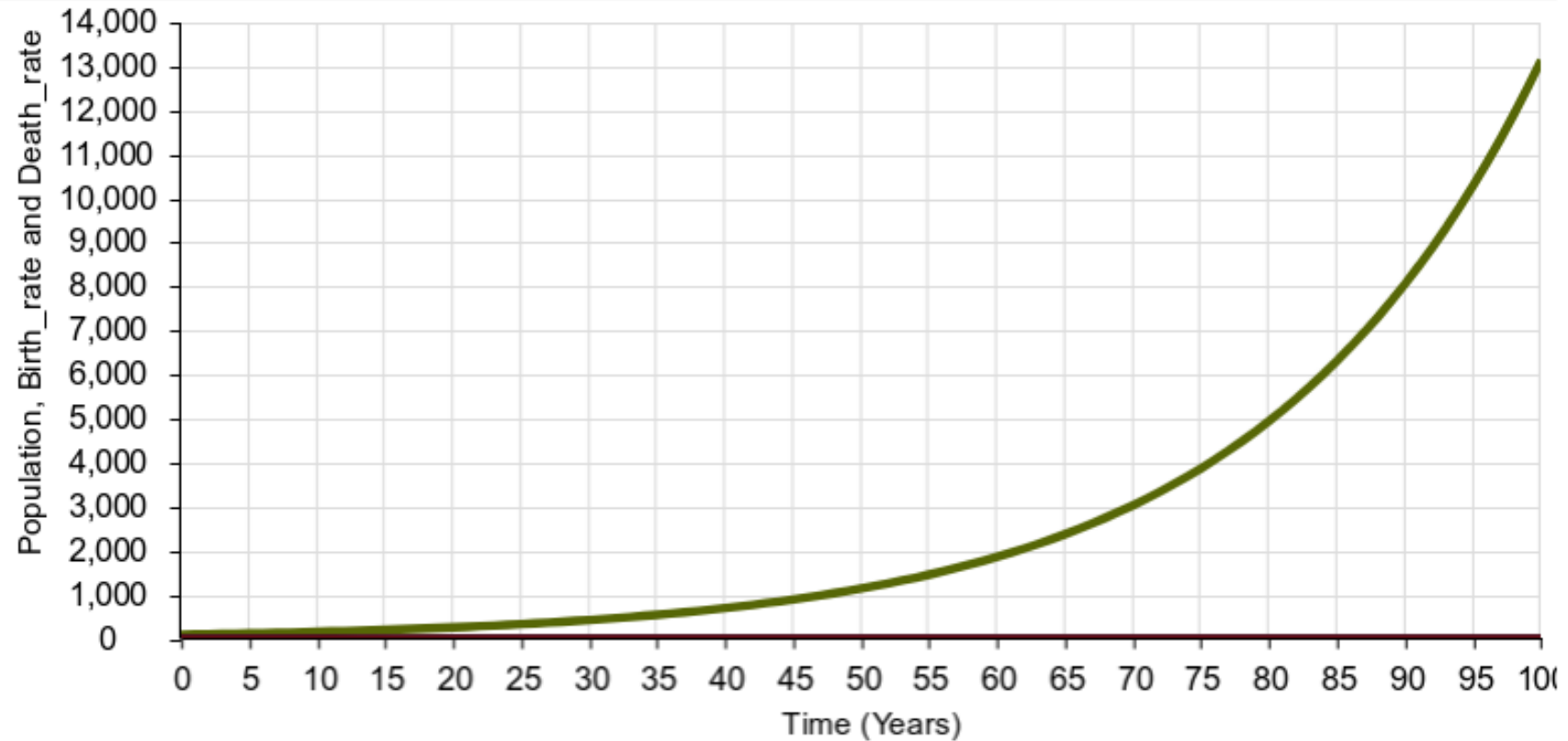
$$\frac{dN}{dt} = rN$$



# Exponential growth



# Exponential growth



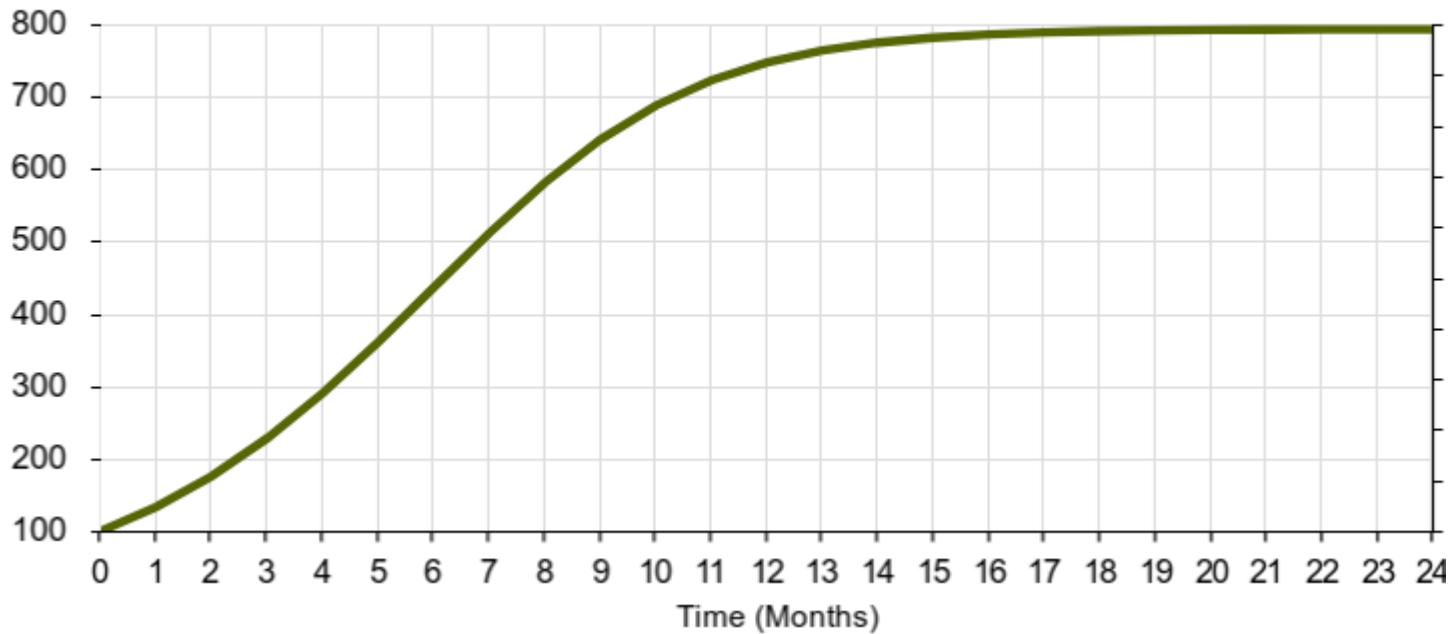
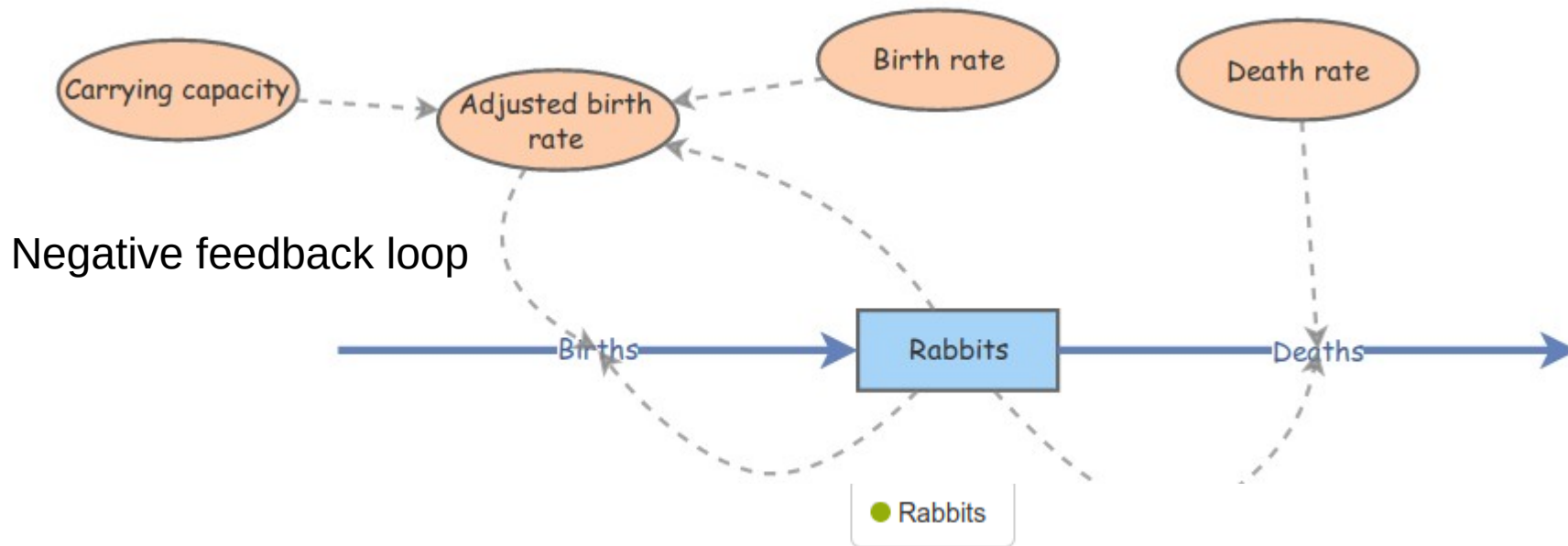
# How do we avoid exponential growth?

We need to include some measure of the carrying capacity in our mathematical model.

As the population approaches the carrying capacity growth should slow to zero.

If the population exceeds the carrying capacity (overshoots) it will decline (negative growth rate)

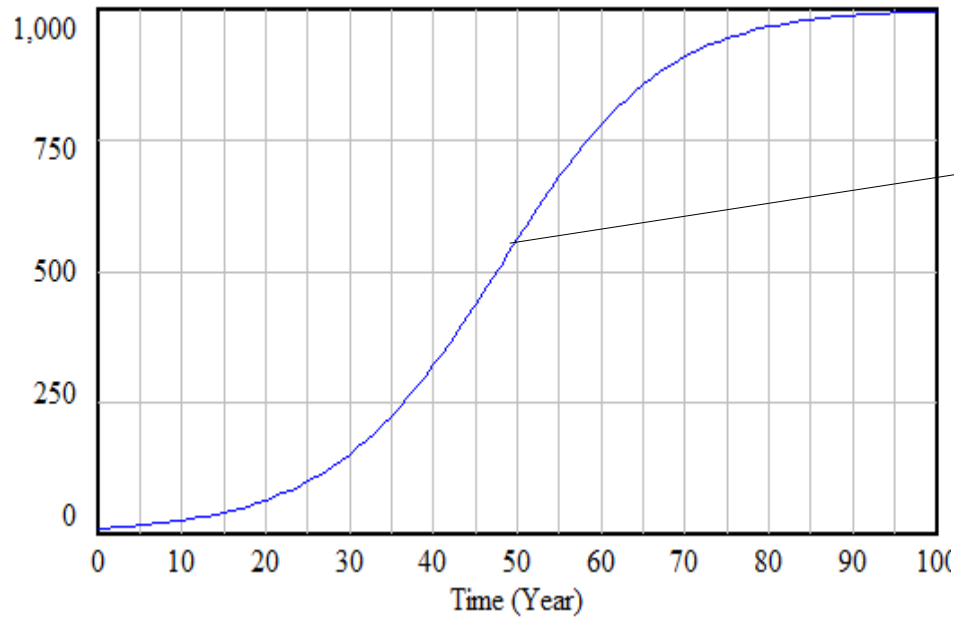
# Adding carrying capacity



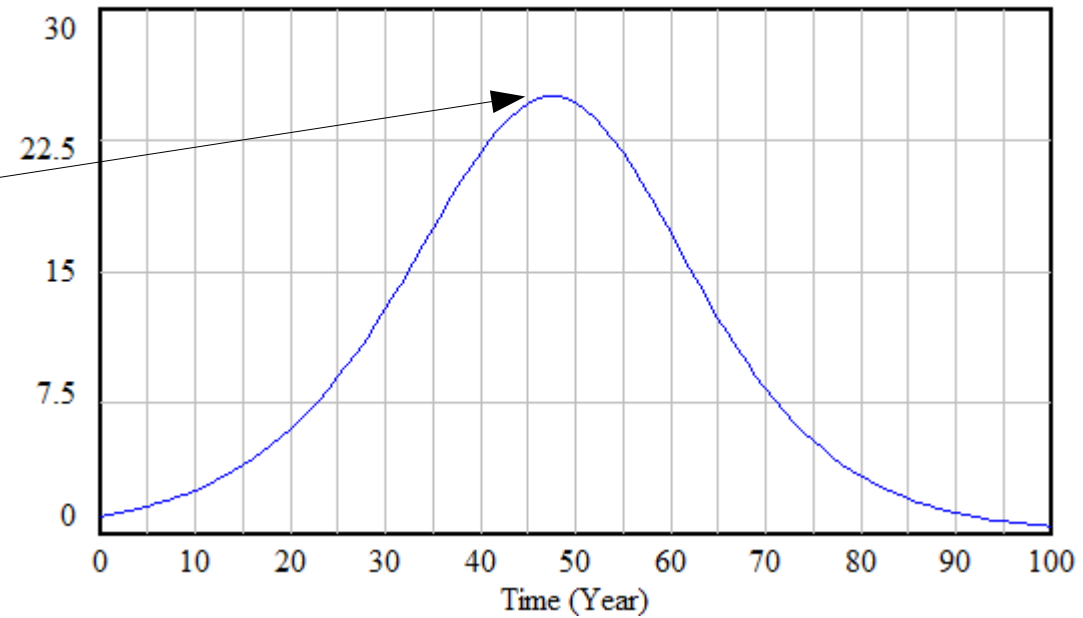
# The logistic equation

$$\frac{dN}{dt} = rN * \left(1 - \frac{N}{K}\right)$$

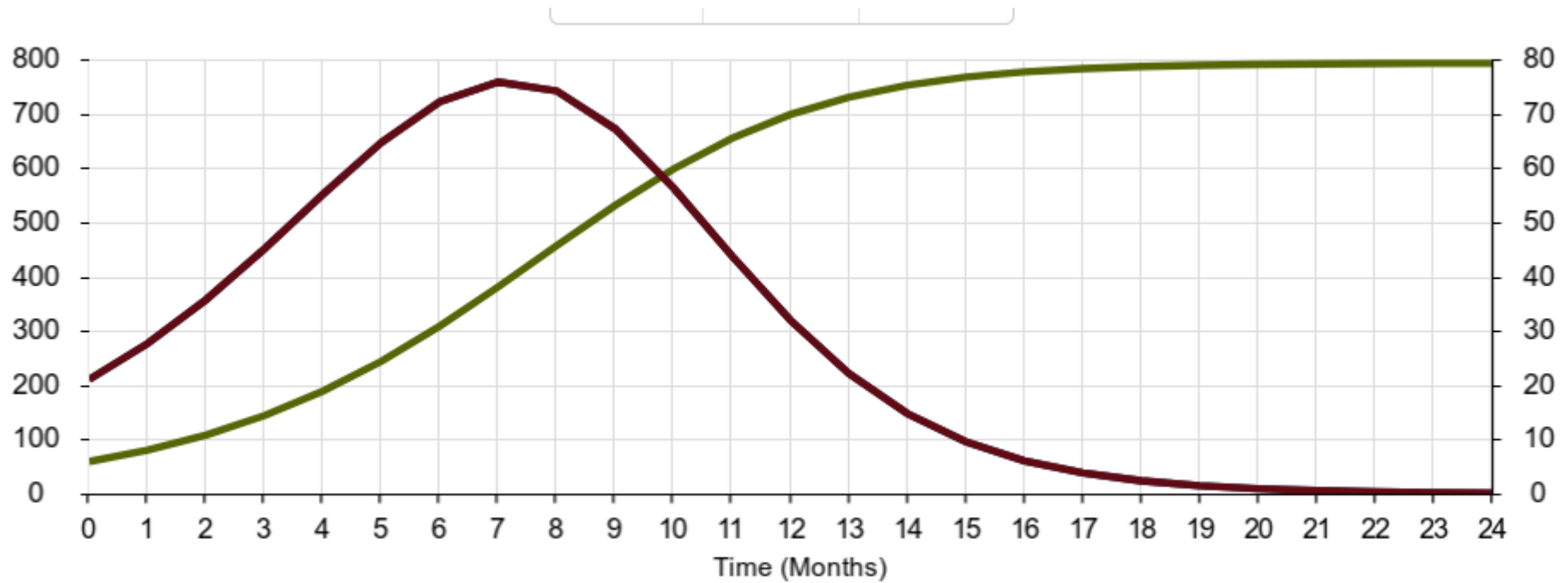
Population



Change



# Logistic equation



# How does this work?

Look at the second term in the equation

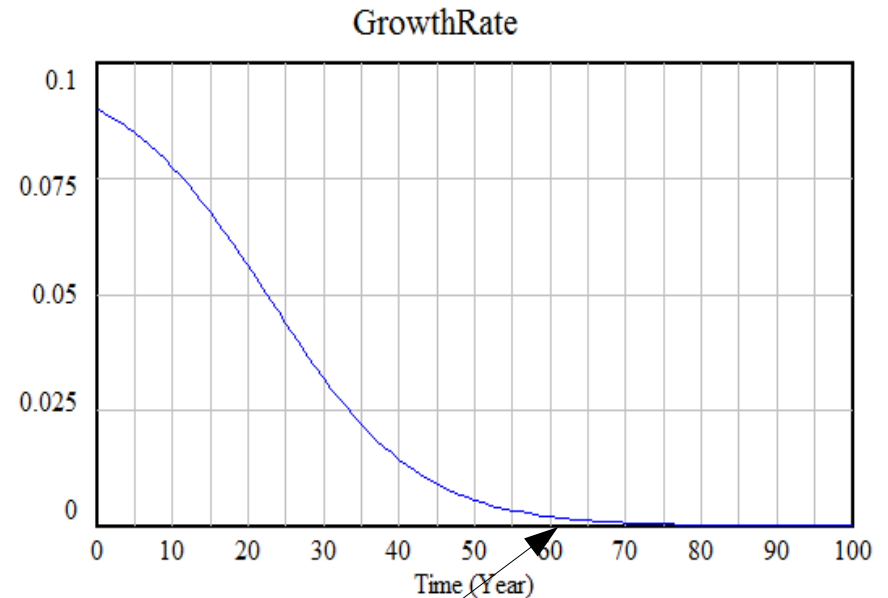
$$\left(1 - \frac{N}{K}\right)$$

If  $N = K$  it becomes  $N/N$

This is equal to one

So  $1 - 1 = \text{zero}$

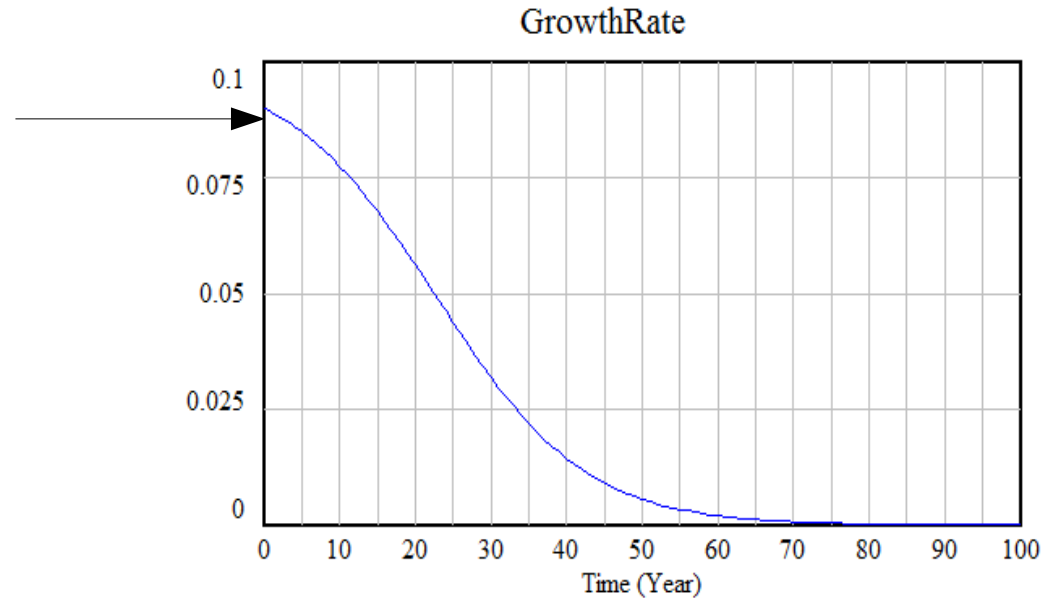
**NO GROWTH at the carrying capacity**





# What if N is small?

$$\left(1 - \frac{N}{K}\right)$$



If N is very small N/K tends to zero

So  $1-0=1$

**So the initial growth rate is not affected  
by the carrying capacity**

# What does this imply?

If we ignore genetic effects (which we shouldn't) populations increase more quickly when they are small.

This is because each individual has greater access to resources

When populations reach the carrying capacity they stop growing

If a population overshoots the carrying capacity it declines

If populations did not tend to “rebound” after declines extinctions would take place more frequently

# What does this imply?

If we manage populations we may wish to hold their size below carrying capacity

“Maximum sustainable yield” may theoretically be obtained by keeping the population around half the carrying capacity.



# R vs K selected species

Populations of large, long lived species may frequently reach and exceed carrying capacity

Natural selection may act on traits that allow survival when competition takes place for resources (K selection)

Small, short lived animals are subject to fluctuations in population size.

Natural selection may act on traits that allow rapid population growth (r selection)

# Cycles and chaos

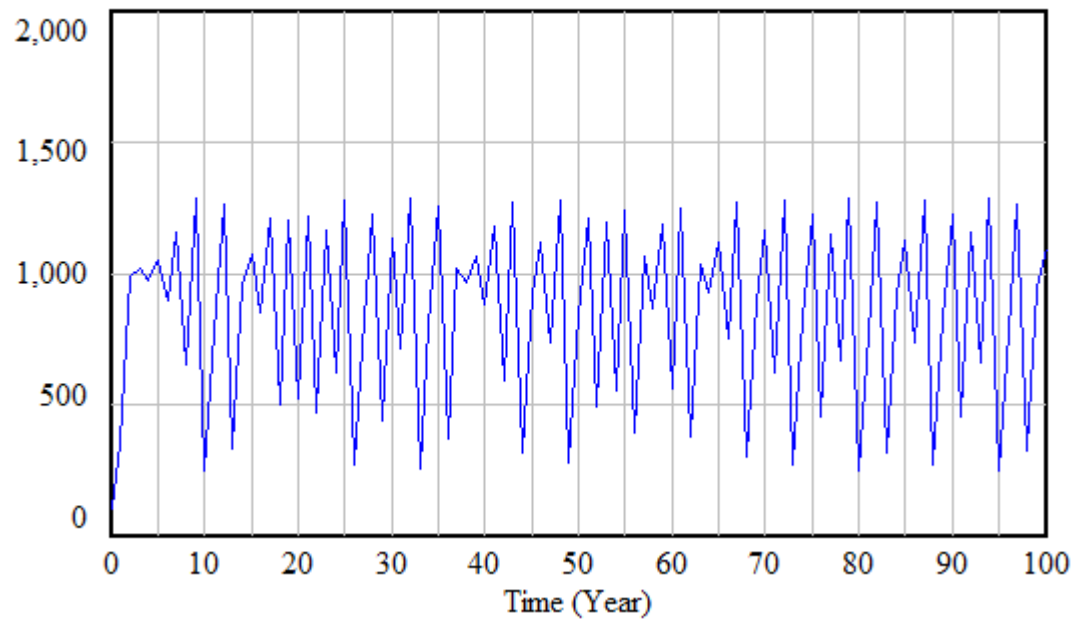
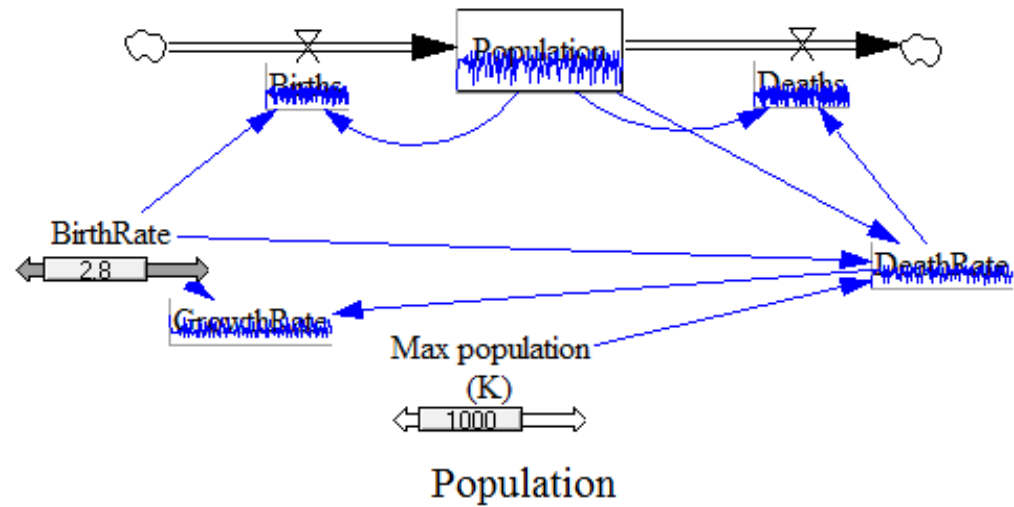
If populations “overshoot” the carrying capacity they will fall

In some circumstances this may produce “boom and bust” cycles

Organisms that are very short lived with high reproductive rates may show chaotic behaviour



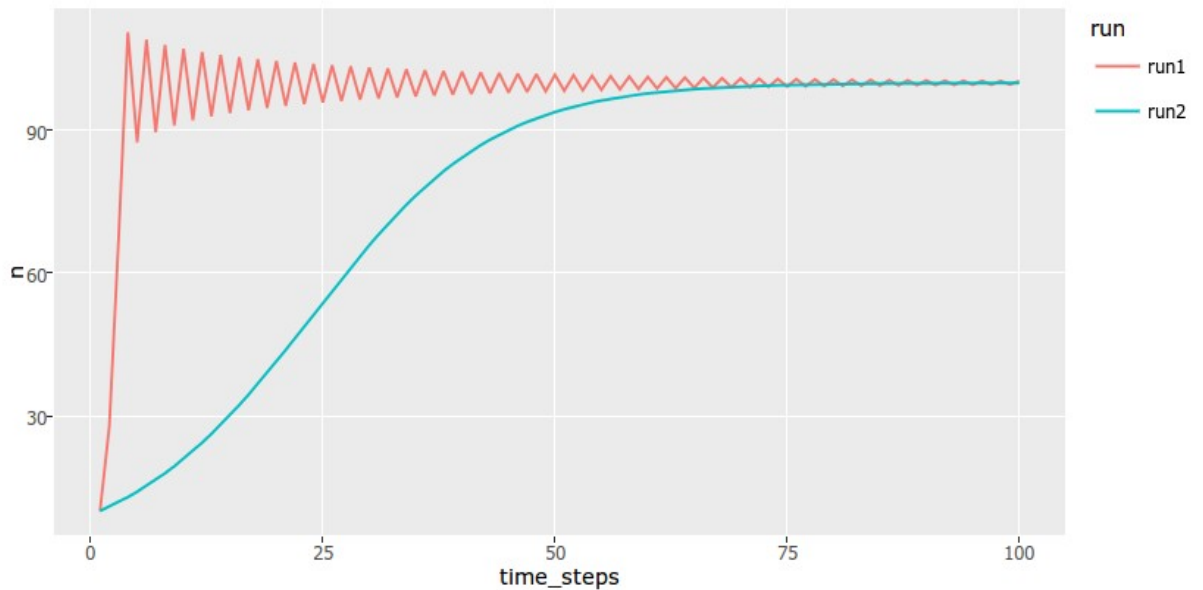
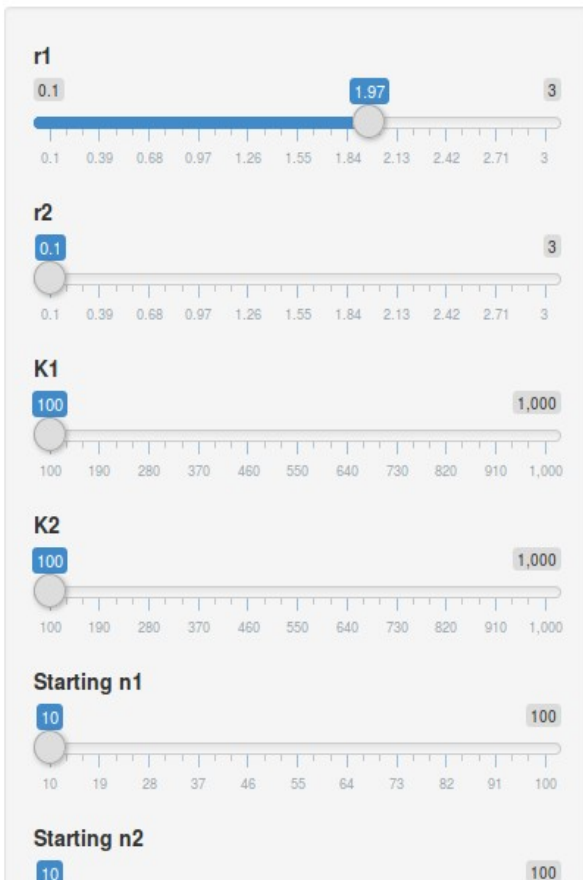
# Chaotic dynamics



# Test it yourself

[https://dgolicher.shinyapps.io/Logistic\\_model](https://dgolicher.shinyapps.io/Logistic_model)

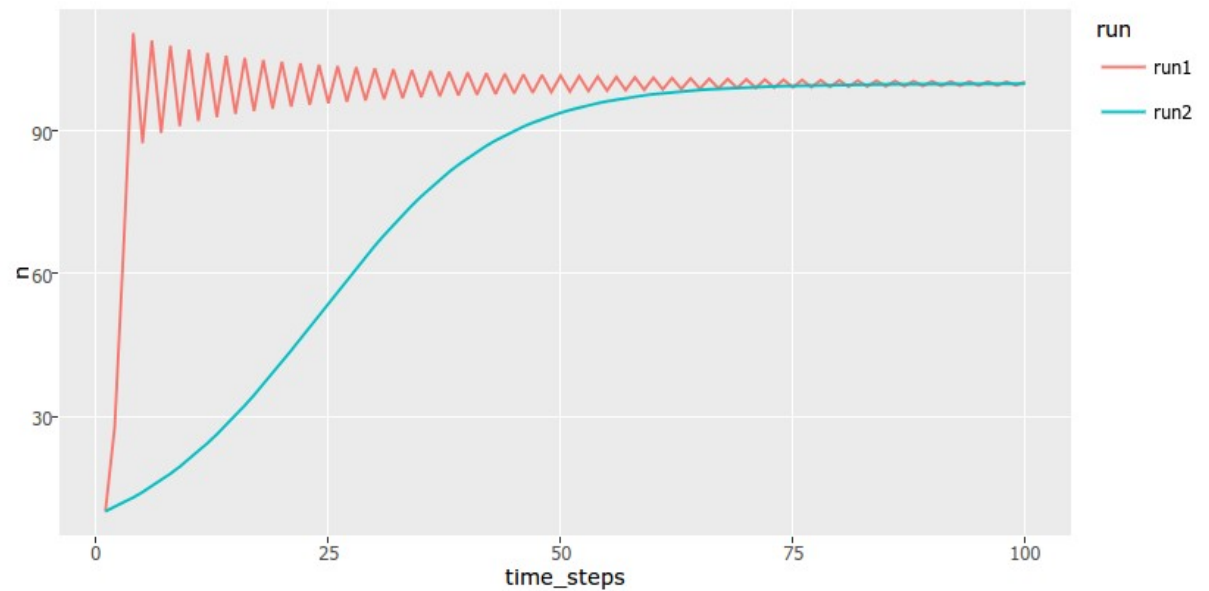
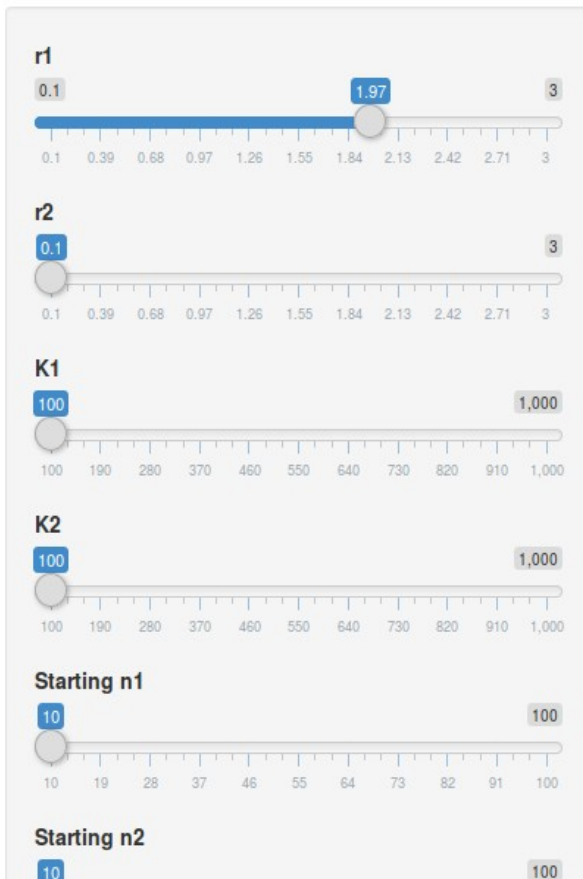
## Logistic model



# Alternative link

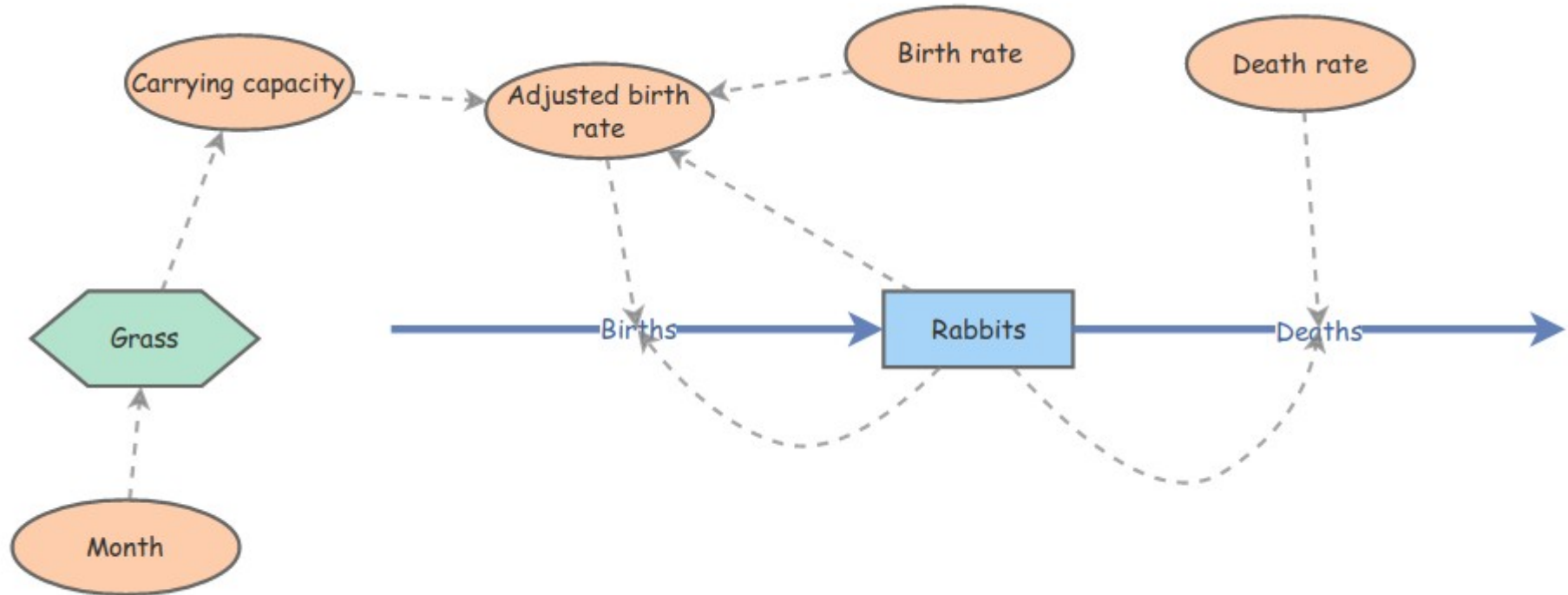
- <http://r.bournemouth.ac.uk:3838/Ecosystems/Logistic/>

## Logistic model

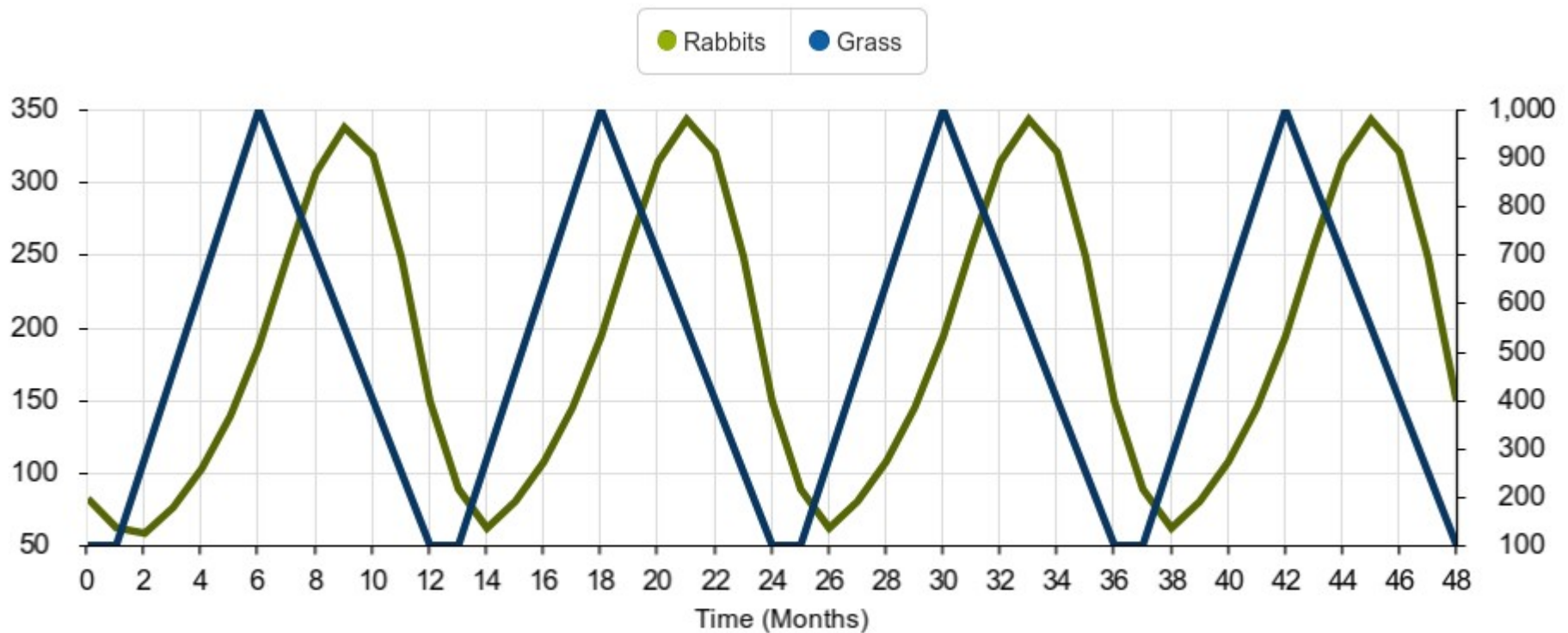




# More realistic models

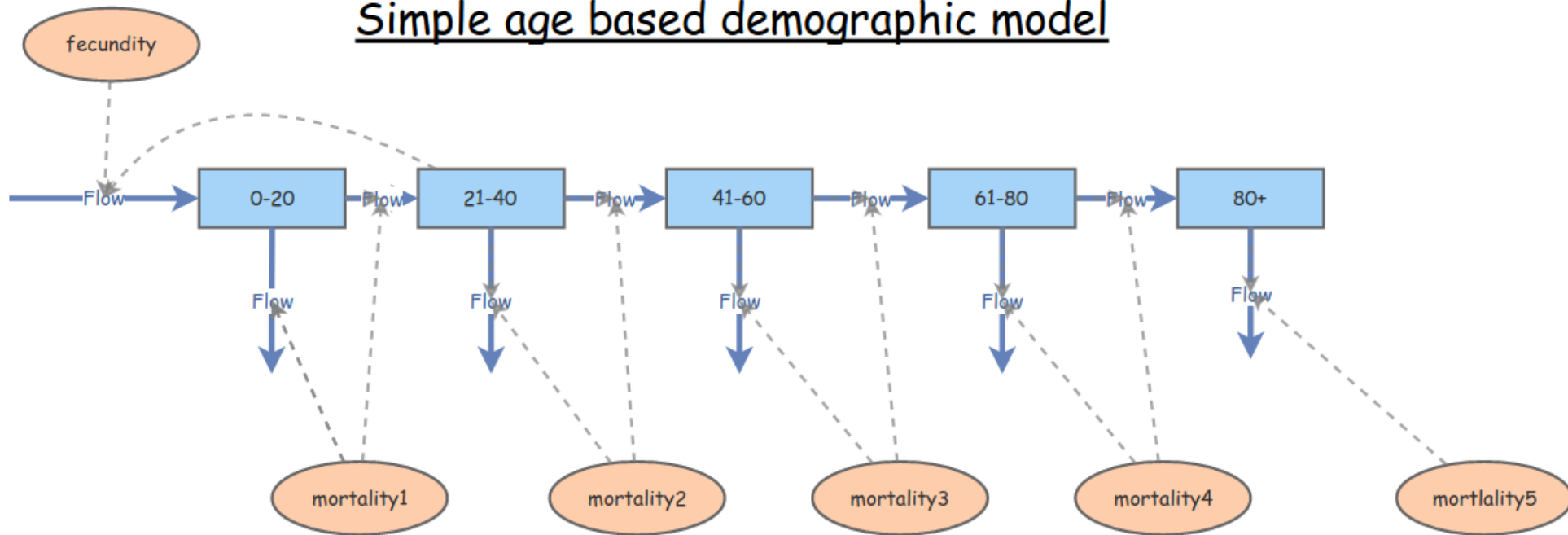


# Carrying capacity varies over year

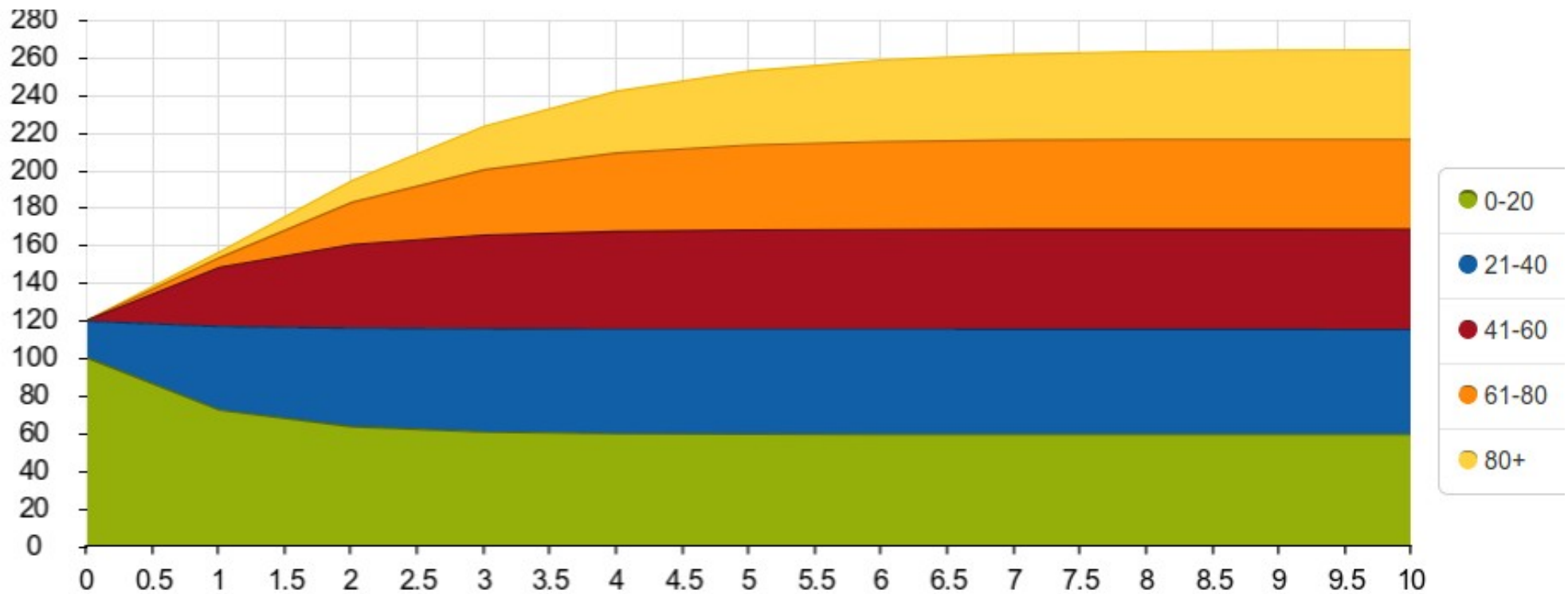


# Age structured model

Simple age based demographic model



# Stable age structure



# Leslie matrix

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t$$

# Eigen values

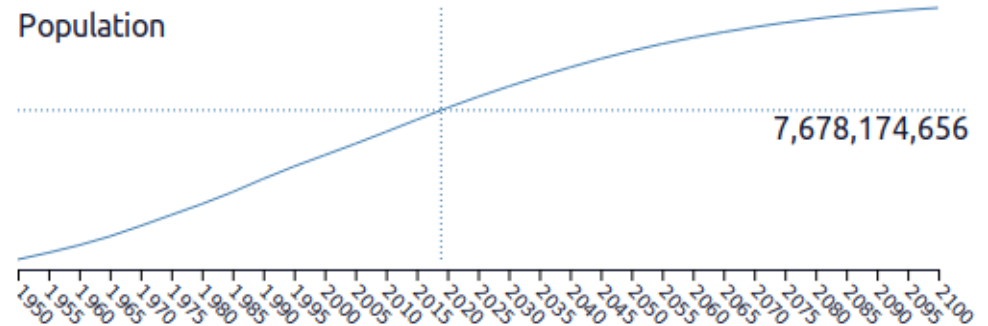
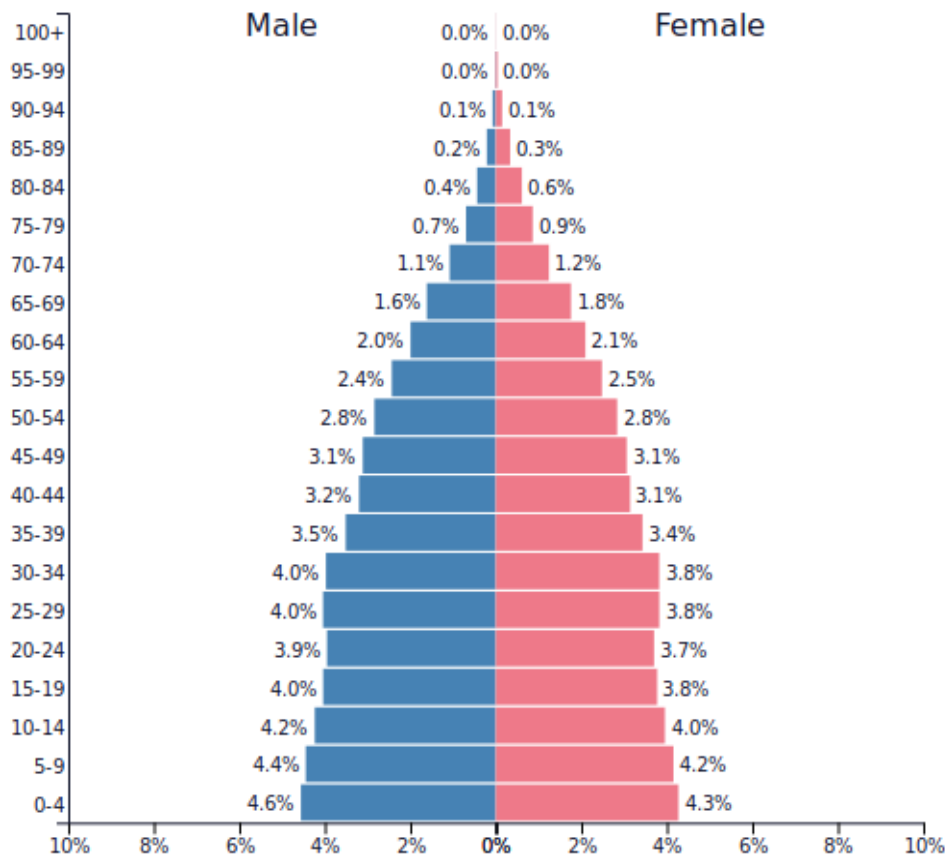
- Steady-state, or stable, age-structure and growth rate.
- Regardless of the initial state the model tends asymptotically to an age-structure and growth rate determined by the matrix
- A population with a high intrinsic growth rate or high mortality will have a “young” age structure (pyramid).
- A population with high survival and low growth will have an “old” age structure

# Population pyramids

- <https://www.populationpyramid.net/world/2019/>

WORLD ▼  
2019

Population: 7,678,174,656



YEAR   2019

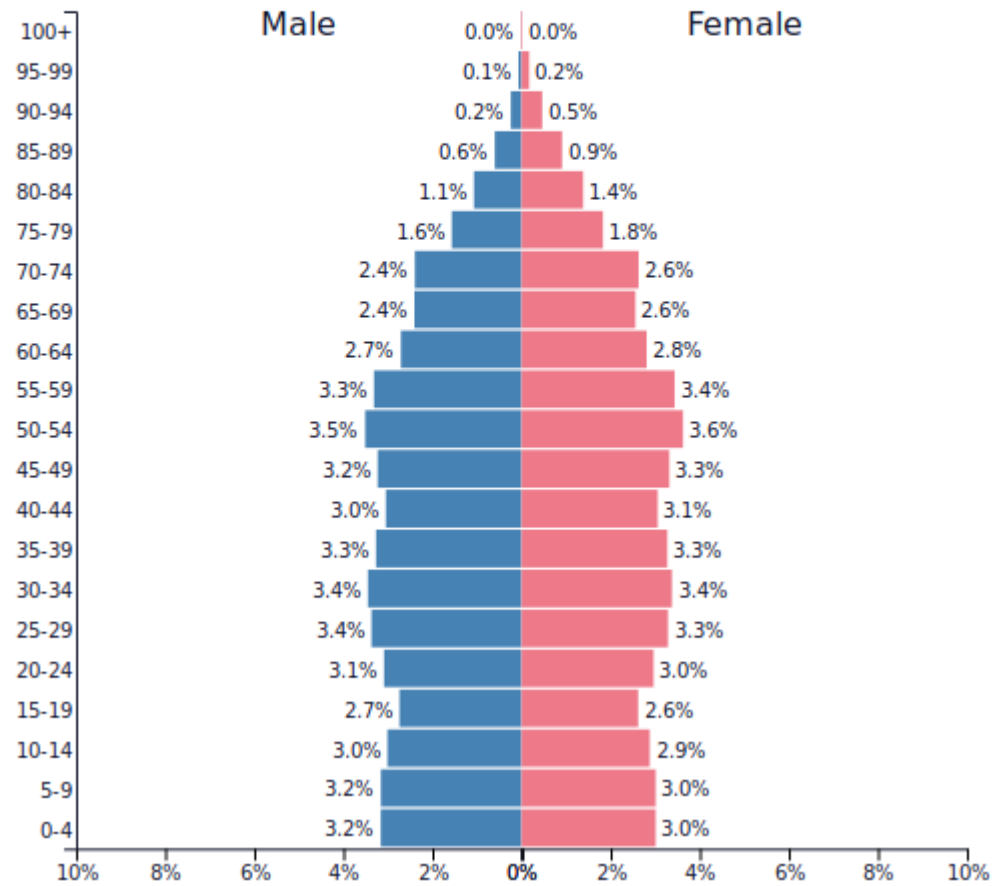
COUNTRY A B C D E F G H I J K L M N O P Q R S T U V W Y Z

Western Africa  
Western Asia  
Western Europe

Western Sahara  
WORLD

# United Kingdom ▼ 2019

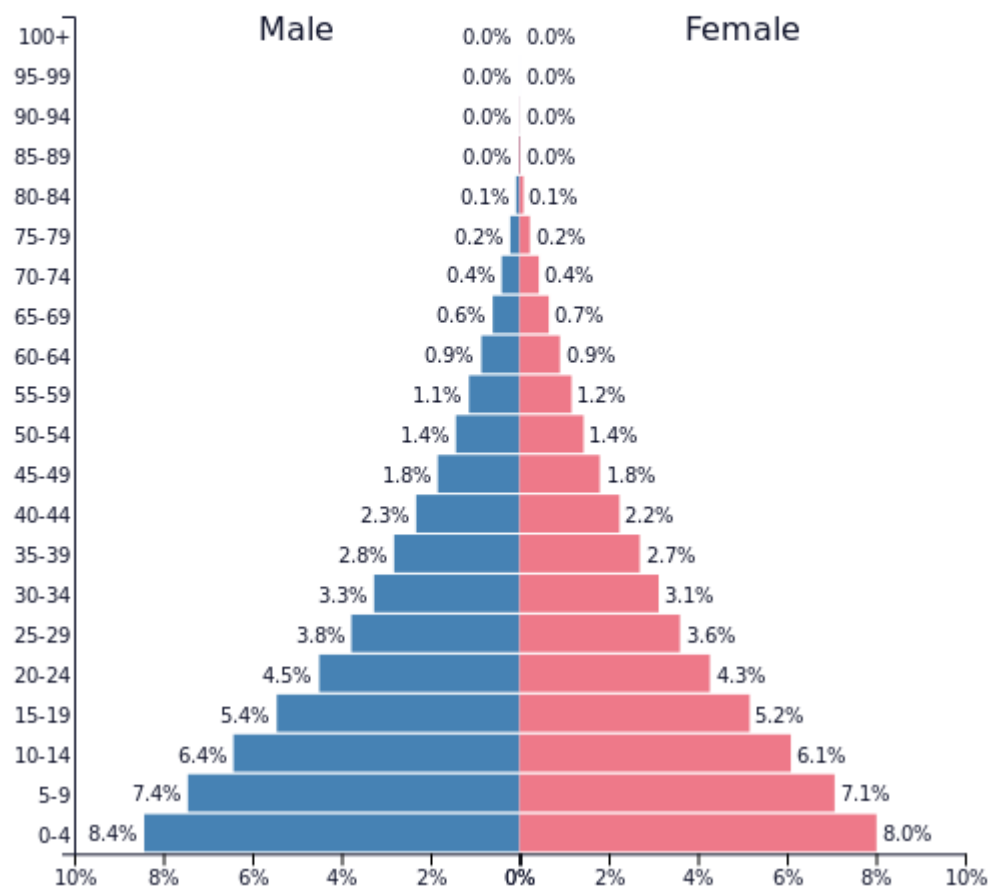
Population: 66,310,254





# Nigeria ▼ 2019

Population: 201,748,560



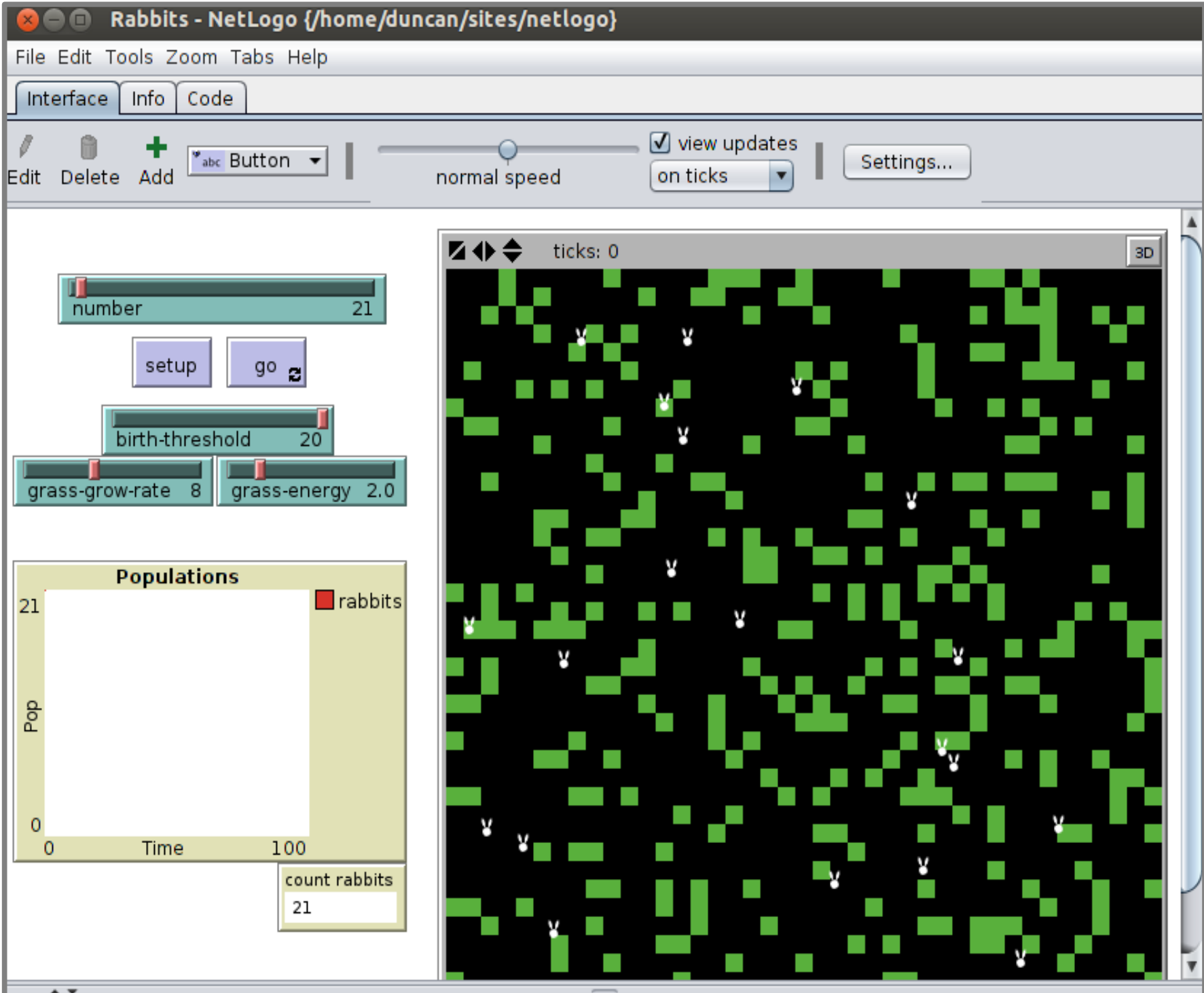
# Individual based model simulation

Use simple rules to investigate system behaviour

Rabbits eat grass to gain energy.

When they obtain enough energy they may reproduce.

The total amount of energy depends on the nutritive value of the grass and the rate of growth after being eaten.



File Edit Tools Zoom Tabs Help

Interface Info Code

Edit Delete Add

Button

normal speed

view updates on ticks

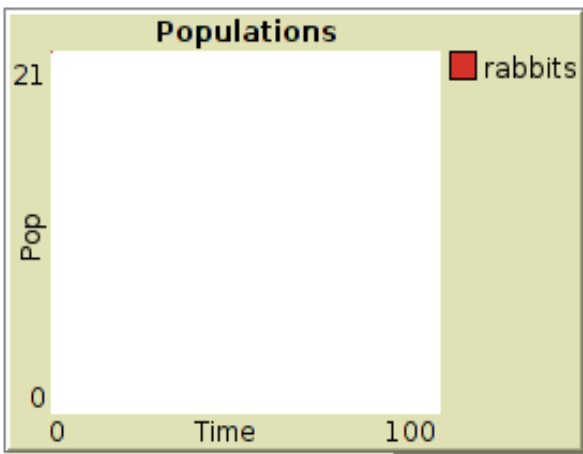
Settings...

number 21

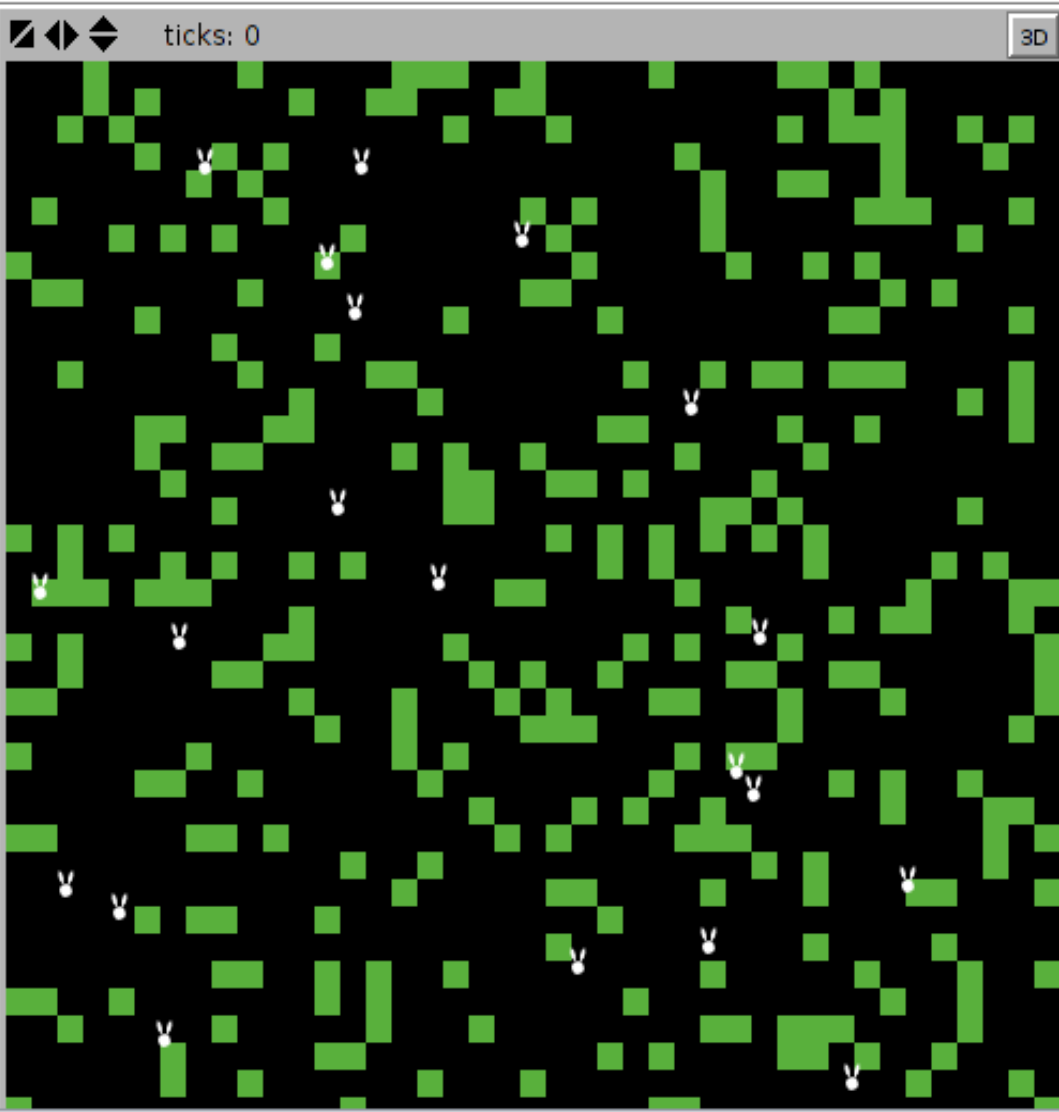
setup go

birth-threshold 20

grass-grow-rate 8 grass-energy 2.0



count rabbits 21



# Link to the model

<http://r.bournemouth.ac.uk:82/Ecosystems/Rabbits%20Grass%20Weeds.html>

# Conclusions

Key words and concepts to take from the last lectures

- Ecosystems. The study of complex, possibly self regulating systems
- System dynamics modelling: Interlinked sets of differential equations that represent complex systems with states that change over time.
- Homeostasis: The tendency of systems to return to a given state
- Positive feedback loops: Processes and linkages that reinforce and accelerate a given trajectory of change
- Negative feedback loops: Processes and linkages that slow down a given trajectory and tend to return a system to an equilibrium state

# Conclusions

Deterministic chaos. Systems that are determined by definable rules but which produce dynamics that are highly sensitive to the initial state and are thus intrinsically unpredictable (butterflies' wing effect